Algebraic Geometry, Warsaw 1960 — 2015
Celebrating honorary degree of the University of Warsaw awarded to Professor Andrzej Białynicki-Birula

Titles and abstracts of talks.

Michel Brion, Institut Fourier, Grenoble
Title: *Local properties of algebraic group actions*
Abstract: Given an algebraic variety X equipped with an action of an algebraic group G, one may ask whether X is covered by open G-stable subsets of a special nature (affine, quasi-projective, subvarieties of projectivizations of G-modules...) When X is normal and G is linear and connected, these questions have been answered positively in classical work of Sumihiro. On the other hand, a nodal cubic curve admits no projective embedding which is equivariant for the action of the multiplicative group. The talk will discuss the general case, where positive answers can be obtained by considering tale coverings.
References: Michel Brion, *On linearization of line bundles*, arXiv:1312.6267 and *On actions of connected algebraic groups*, arXiv:1412.1906 .

Hélène Esnault, Freie Universität, Berlin
Title: *Convergent isocrystals on simply connected varieties*
Abstract: The talk will report on joint work with Atsushi Shiho. We show that under an additional assumption, if X is smooth projective simply connected over an algebraically closed field of characteristic $p > 0$, then convergent isocrystals are trivial. This answers positively, under this extra assumption, a conjecture by Johan de Jong.

Vladimir Popov, Steklov Institute, Moscow
Title: *Algebraic subgroups of the Cremona groups*
Abstract: The first results on the tori in the Cremona groups were obtained in the 60s of the last century by A. Białynicki-Birula. The talk is aimed at describing some later advances in understanding algebraical subgroups of the Cremona groups, including diagonalizable subgroups and triangulable subgroups, and the related problems like the Abhyankar–Sathaye conjecture, and the Bass' question.

Andrzej Schinzel, Mathematical Institute, Polish Academy of Sciences
Title: *On the congruence* $f(x) + g(y) + c = 0$ (mod xy)
Abstract: L. J. Mordell in his paper in Acta Math. 44 (1952) stated the following theorem. If $f(x) = a_0x^m + a_1x^{m-1} + ... + a_{m-1}x$, $g(y) = b_0y^n + b_1y^{n-1} + ... + b_{n-1}y$, a_i, $(0 \leq i < m)$, b_j $(0 \leq j < n)$ and c are integers, then the congruence $f(x) + g(y) + c = 0$ (mod xy) has infinitely many solutions in integers x, y. He outlined a proof for $f(x) = ax^3$, $g(y) = by^3$, $abc \neq 0$. In the talk we study Mordell's assertion, which is not true in general, but under additional assumption.

Andrzej Skowroński, Nicolaus Copernicus University, Toruń
Title: *Periodic Algebras*
Abstract: A finite dimensional algebra A (associative, with identity) over an algebraically closed field K is called a periodic algebra if it is periodic under the action of the syzygy operator in the category of finite dimensional $A - A$-bimodules. Periodic algebras are selfinjective, have periodic module categories, periodic Hochschild cohomology, and are invariant under derived equivalences. Periodic algebras have also interesting connections with group theory, topology, singularity theory, invariant theory, and cluster algebras. The classification of all periodic algebras (up to Morita equivalence) may be in reach, and is one of the most attractive open problems of the current representation theory of finite dimensional algebras. We will present distinguished classes of periodic algebras and show connections of periodic algebras with finite groups, hypersurface singularities and triangulated surfaces.

Andrew Sommese, University of Notre Dame
Title: *Numerical Algebraic Geometry*
Abstract: The goal of Numerical Algebraic Geometry is to carry out algebraic geometric calculations in characteristic zero using numerical analysis algorithms. This comes down to numerical algorithms to compute and manipulate solution sets of polynomial systems. Numerical Algebraic Geometry is a natural outgrowth of the continuation methods to compute isolated complex solutions of systems of polynomials with complex coefficients. There is a wide range of applications including the solution of chemical systems, kinematics, the numerical solution of systems of nonlinear differential equations, theoretical physics, the computation of algebraic geometric invariants, ...
Bertini is open-source C software, developed by Bates (Colorado State U.), Hauenstein (Notre Dame), Sommese (Notre Dame), and Wampler (General Motors R. & D.), to carry out Numerical Algebraic Geometry computations.

Bertini will be rewritten to make it a better tool for users. Bertini dates from over a decade ago, and from this experience we have identified several possibilities for significant improvements. One goal is to change some of the data structures and add internal functionality that will give the user the ability to write scripts and interface with other software.
In this talk, I will give an overview of Numerical Algebraic Geometry with an especial focus on applications to the numerical solution of systems of nonlinear differential equations. I will consider the theoretical algorithms underlying the area in the light of the practical issues that arise when implementing the algorithms in the current and the future Bertini.

Jerzy Weyman, University of Connecticut
Title: *Semi-invariants of quivers, cluster algebras and the hive model*
Abstract: The saturation theorem for Littlewood-Richardson coefficients was a fashionable subject about a decade ago. There are two completely different proofs of the theorem: the original one by Knutson-Tao based on their hive model, and a proof based on quiver representations given by Harm Derksen and myself. So far there was no link between these two proofs.
Recently Jiarui Fei discovered a remarkable cluster algebra structure on the ring $SI(T_{n,n,n}, \beta(n))$ of semi-invariants of a triple flag quiver, whose weight spaces have dimensions that are Littlewood-Richardson coefficients.
In proving his result he uses both the hive model and the quiver representations. It turns out that the link between the two approaches is the quiver with potential underlying the cluster algebra structure. The combinatorics of g-vectors for this quiver with potential turns out to be identical to the hive model.
In my talk I will explain the notions involved and basic ideas behind Jiarui Fei's proof.

Jarosław Włodarczyk, Purdue University, West Lafayette
Title: *Generalized implicit function theorem and singularities.*
Abstract: Building upon ideas of Hironaka and Bierstone-Milman we generalize the implicit function Theorem (in differentiable, analytic and algebraic situation) for the sets of functions of larger multiplicities (or ideals). It allows to describe singularities given by a finite set of generators or by ideals in a simpler form. As a consequence we prove a Cohen-Macaulay analog of the inverse function theorem.
The generalized implicit function theorem is closely related to a proven here extended version of Weierstrass-Hironaka division and preparation theorems.

The results are valid in algebraic, analytic, and differentiable setting, and rely, in particular, on Malgrange preparation theorem for modules .
We show some applications of the theorems to desingularization (Hironaka normal flatness, existence of weak standard basis for ideals and description of Samuel stratum). One of the tools used in the proofs is, introduced here, filtered Stanley's decomposition for the graded rings.